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SOLUTIONS.

2897 [1921, 228]. Proposed by PAUL CAPRON, U. S. Naval Academy.

Discuss the conditions under which the angles made by two circles on a sphere have the same measures as the distances between their poles.

SOLUTION BY F. L. WILMER, Omaha, Neb.

Write the equations of the circles:

$$\left.\begin{matrix} x^2+y^2+z^2=1 \\ x\cos\phi+y\sin\phi=q \end{matrix}\right\},\qquad \left.\begin{matrix} x^2+y^2+z^2=1 \\ x=p \end{matrix}\right\},$$

where ϕ is the lesser angular distance between their poles. If (x', y', z') is the point of intersection for which z' is positive, then

$$x'=p,\qquad y'=\frac{q-p\cos\phi}{\sin\phi},\qquad z'=\frac{1}{\sin\phi}\sqrt{\sin^2\phi-d^2},$$

where d is the distance between the centers of the two circles. The length of the subnormal at (x', y', z') is $\dfrac{q-p\cos\phi}{\sin\phi}$ for the circle of radius $\sqrt{1-p^2}$, and $\dfrac{p-q\cos\phi}{\sin\phi}$ for the circle of radius $\sqrt{1-q^2}$.

The direction cosines are then:

$$0,\qquad -\frac{\sqrt{\sin^2\phi-d^2}}{\sin\phi\sqrt{1-p^2}},\qquad \frac{q-p\cos\phi}{\sin\phi\sqrt{1-p^2}}$$

for the one, and

$$-\frac{\sqrt{\sin^2\phi-d^2}}{\sqrt{1-q^2}},\qquad \frac{\cos\phi\sqrt{\sin^2\phi-d^2}}{\sin\phi\sqrt{1-q^2}},\qquad \frac{p-q\cos\phi}{\sin\phi\sqrt{1-q^2}}$$

for the other tangent line at (x', y', z').

The proposition requires:

$$-\frac{\cos\phi(\sin^2\phi-d^2)}{\sin^2\phi\sqrt{(1-p^2)(1-q^2)}}+\frac{(q-p\cos\phi)(p-q\cos\phi)}{\sin^2\phi\sqrt{(1-p^2)(1-q^2)}}=\cos\phi.$$

This reduces to

$$\frac{qp-\cos\phi}{\sqrt{(1-p^2)(1-q^2)}}=\cos\phi=\frac{qp}{1+\sqrt{(1-p^2)(1-q^2)}}.$$

NOTE BY THE EDITORS—This equation may also be obtained by considering the stereographic projection of the sphere upon the xy-plane. Circles on the sphere project into circles in the plane and angles are preserved by the projection. The equations of the two circles are

$$x^2+y^2-(2/p)x+1=0 \text{ and } x^2+y^2-(2/q)(x\cos\varphi+y\sin\varphi)+1=0,$$

and for the cosine of the angle between them we find the same expression as that given in the solution above.

Another way of treating the problem is by spherical trigonometry. The two poles on the sphere and the intersection of the two circles are the vertices of a spherical triangle in which the measure of one angle is equal or supplementary to the measure of the opposite side. These conditions are satisfied when a triangle has two right angles. In other cases the cosine law and the cosine law for the polar triangle, to give consistent results, require that the supplementary relation hold for all three angles and their opposite sides, or only for one. On the other hand, at least two angles of a spherical triangle are in the same quadrants as their opposite sides. Therefore, unless our triangle has two right angles, it must have one angle supplementary and each of the other two equal to their opposite sides. The side whose opposite angle is supplementary is the one which is nearest to 90°.[1] The equation obtained above is merely the cosine law of spherical trigonometry.

2901 [1921, 277]. Proposed by NATHAN ALTSHILLER-COURT, University of Oklahoma.

Given the length of the base of a variable triangle and the positions of the feet of the altitudes on the other two sides, find (*a*) the locus of the vertex opposite the base; (*b*) the locus of the foot of the altitude on the base.

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[1] See Chauvenet, *Plane and Spherical Trigonometry*, edition 9, Philadelphia, 1881, p. 178.